[13.10] Let G and H be groups.

(a) Verify that G×H is a group where G×H is the set { (*g*, *h*) } with the operation 

1. Show that we can identify H with G×H /G
2. (1,1) is the identity since (1,1) (*g*,*h*) = (*g*,*h*) = (*g*, *h*) (1,1) ✔

The inverse of (*g*,*h*) is (g-1,*h*-1) since  ✔

The associative law holds:

  ✔

Therefore G×H is a group. ✔

1. There are 3 preliminaries to cover.
   1. G must be a subgroup of G×H. It isn’t, but it can be identified with  which *is* a subgroup. We henceforth use the symbol G to represent group G = { g } as well as group G×{1} = { (*g*,1) : *g* ∈G }, and it should always be obvious which is meant.
   2. G×H /G Is a group only if G is normal in G×H. In order to show that G is normal we must show that 

Fix (*g*,*h*) ∈ G×H. For any *g*1 ∈ G,

To show equality, let *g*2 ∈ G. We need to find a *g*1 ∈ G such that 

Define  Then  ✔

* 1. We require the fact that *g*G = G for any *g* ∈ G:

Fix *g*. Clearly *g* G ⊆ G (since *g* *g*1 ∈ G for any *g*1 ∈ G). To show equality, let *g*1 ∈ G. We want to find a *g*2 such that  Define *g*2 = *g*-1 *g*1. Then indeed  ✔

We are now ready to show the identification of H with G×H /G. By definition G×H /G = { G(g,h) }. Since *g* G = G,  So for each coset G (*g*,*h*) we can choose any element of G to be the representative element. Choose *g* = 1. That is, G×H /G = { G(1,*h*) ) : *h* ∈H }, and we readily identify H = { *h* ∈H } with { G(1,*h*) : *h* ∈H } = G×H /G. In fact we have shown that H is group isomorphic to G×H /G. ✔