[13.10] Let G and H be groups.

(a) Verify that G×H is a group where G×H is the set { (*g*, *h*) } with the operation 

1. Show that we can identify H with (G×H) /G
2. (1,1) is the identity since (1,1) (*g*,*h*) = (*g*,*h*) = (*g*, *h*) (1,1) ✔

The inverse of (*g*,*h*) is (g-1,*h*-1) since  ✔

The associative law holds:

  ✔

Therefore G×H is a group. ✔

1. Recall that (G×H) /G = {G (*g*,*h*): (*g*,*h*)∈G×H }.

There are 2 preliminaries to cover.

* 1. G must be a subgroup of G×H. It isn’t, but it can be identified with  which *is* a subgroup.
  2. (G×H) /G\* is a group only if G\* is normal in G×H. In order to show that G\* is normal we must show that

Fix (*g*,*h*) ∈ G×H. For any *g*1 ∈ G, 

(*g*,1) = (1,1) (*g*,1) (1-1,1-1) G\* (*g*,*h*) G\* (*g*-1,*h*-1).



Therefore G\* = (*g*,*h*) G\* (*g*-1,*h*-1) ✔

We now show that H can be identified with (G×H) /G\* by providing an isomorphism. Define

*p* : H  (G×H) /G\* : *p*(*h*) = G\* (1,*h*).

We show that *p* is a group homomorphism that is 1-1 and onto.

Homomorphism:

Since GG = G, then G\* G\* = G\*, so

*p*(*h*1 *h*2) = G\* (1,*h*1 *h*2) = G\* G\* (1, *h*1) (1, *h*2) = G\* (1, *h*1) G\* (1, *h*2)

= *p*(*h*1) *p*(*h*2). ✔

Onto

Let G\* (1,*h*)  (G×H) /G\*. Then *p*(*h*) = G\* (1,*h*). ✔

1-1

If *h*1 ≠ *h*2 then G\* (1,*h*1) = {(*g*, *h*1) : *g*∈G} ≠ {(*g*, *h*2) : *g*∈G} = G\* (1, *h*2)  